Signals and Systems

Vlcinfo

Visvesvaraya Learning Center

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Chapter 1: Introduction

Signals and systems theory provides a framework for understanding and manipulating signals and the systems that process them, with applications in various fields including telecommunications, control systems, audio processing, image processing, and more.

Signal

- A signal is any physical quantity that varies with time, space, or any other independent variable. Signals can represent various phenomena such as sound, images, temperature, voltage, etc. In the context of electrical engineering, signals are often represented as functions of time.

System

- A system is any physical device or mathematical operation that processes an input signal to produce an output signal. Systems can be classified based on their behavior, such as linear or nonlinear, time-invariant or time-varying, causal or non-causal, etc.

Signal Processing

- Signal processing involves analyzing, modifying, and synthesizing signals to extract useful information or to achieve a specific objective. It encompasses a wide range of techniques, including filtering, modulation, sampling, and transformation.

System Analysis

- System analysis deals with understanding the behavior of systems in response to different input signals. This includes studying properties like stability, linearity, causality, and frequency response.


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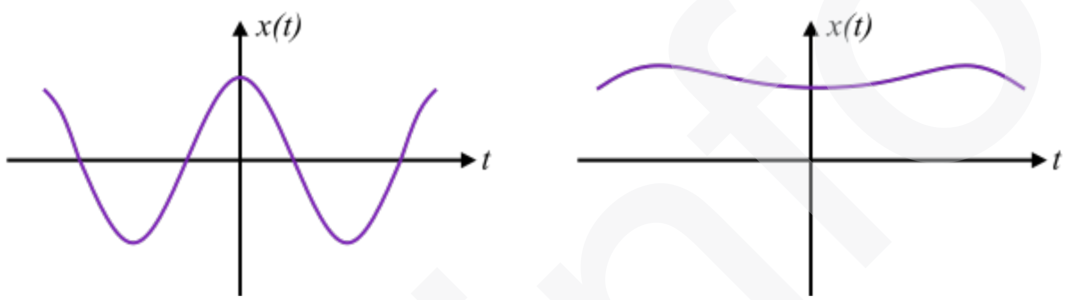
Chapter 2: Classification of signals

Depending upon the nature and characteristics in the time domain, the signals may be classified into two categories,

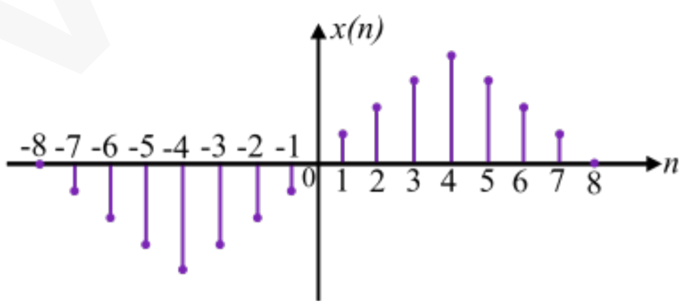
1. Continuous Time Signals

- Continuous-time signals are mathematical functions that vary continuously with respect to time. In simpler terms, they are signals that exist and change continuously over a continuous range of time.
- In mathematical terms, a continuous-time signal x(t) is defined for all real values of time t within a specified interval (-∞,∞) or [a, b], where a and b are real numbers.



2. Discrete Time Signals

- Discrete-time signals are mathematical sequences of values that are defined only at discrete points in time.
- Discrete-time signals are representations of physical phenomena or information that are observed or measured only at specific, discrete instances of time.
- In mathematical terms, a discrete-time signal x[n] is defined only for integer values of the index n, where n typically represents time instances such as samples, intervals, or events.



Chapter 3: Classification of Systems

Systems can be classified based on various attributes including their behavior, characteristics, and mathematical representations. Here's a classification of systems:

1. Based on Behavior:

Linear Systems:

- Follow the principle of superposition and homogeneity.
- The output of a linear system to a sum of inputs is equal to the sum of the outputs of each input applied separately.

Nonlinear Systems:

- Do not follow the principles of superposition and homogeneity.
- The output is not directly proportional to the input.

2. Based on Time Invariance:

Time-Invariant Systems:

- The system characteristics and behavior do not change over time.
- The response of the system to a signal remains the same regardless of when the signal is applied.

Time-Varying Systems:

- The system characteristics and behavior change over time.
- The response of the system to a signal depends on when the signal is applied.

3. Based on Causality:

Causal Systems:

- The output depends only on present and past values of the input.
- Future values of the input do not affect the current output.

Non-Causal Systems:

- The output depends on future values of the input or has memory elements.
- The output may depend on future values of the input, making it impossible to implement in real-time systems.

4. Based on Stability:

Stable Systems:

- Bounded input signals produce bounded output signals.
- The output does not exhibit unbounded growth or oscillations for bounded inputs.

Unstable Systems:

- Bounded input signals produce unbounded output signals.
- The system's response grows without bound or exhibits oscillations for bounded inputs.

5. Based on Linearity and Time-Invariance (LTI):

Linear Time-Invariant (LTI) Systems:

- Satisfy both linearity and time-invariance properties.
- Widely studied due to their mathematical tractability and practical importance.

Chapter 4: Behavior of Systems

A. Behavior of Continuous-Time (CT) LTI Systems

1. Linearity:

- A continuous-time LTI system is linear if it satisfies the principle of superposition. That is, if x1 (t) produces y1(t) and x2(t) produces y2 (t), then a1x1(t) + a2x2(t) produces a1y1(t) + a2y2(t), where a1 and a2 are constants.

2. Time-Invariance:

- A continuous-time LTI system is time-invariant if its response to a shifted input x(t) is a shifted version of its response to x(t). In other words, if y(t) is the response to x(t), then y(t) is the response to x(t-τ).

B. Behavior of Discrete-Time (DT) LTI Systems

1. Linearity:

- A discrete-time LTI system is linear if it satisfies the principle of superposition. That is, if x1 [n] produces y1 [n] and 22 [n] produces y2 [n], then a1x1 [n] + a2x2 [n] produces a1y1 [n] + a2y2[n], where a1 and a_2 are constants.

2. Time-Invariance:

- A discrete-time LTI system is time-invariant if its response to a shifted input x[nk] is a shifted version of its response to x[n]. In other words, if y [n] is the response to x [n], then y[nk] is the response to x[nk].

Note :

- Both CT and DT LTI systems exhibit superposition, meaning that their response to a linear combination of inputs is the same as the linear combination of their responses to individual inputs.
- Both CT and DT LTI systems are time-invariant, meaning that their response to a shifted input signal is a shifted version of their response to the original signal.

Chapter 5: Linear Time-Invariant (LTI) Systems

Definition of LTI systems

- An LTI system is a system whose output response to an input signal is linearly related to the input signal and is not dependent on the absolute time reference but rather on the relative time difference between the input and output.
- Satisfy both linearity and time-invariance properties.
- Widely studied due to their mathematical tractability and practical importance.

Properties of LTI systems

1) ***Linear:*** $k \cdot x(t) * h(t) = k \cdot (x(t) * h(t))$

2) ***commutative:*** $x(t) * h(t) = h(t) * x(t)$

3) ***associative:*** $[x(t) * h(t)] * w(t) = x(t) * [h(t) * w(t)]$

4) ***distributive:*** $x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$

Chapter 6: Fourier Series

Definition

Fourier Series is a powerful mathematical tool used to analyze and represent periodic signals. The Fourier Series represents a periodic function f(t) as a sum of sinusoidal functions (sine and cosine) or complex exponentials.

Mathematical Representation

If $f(t)$ is a periodic function with period T, its Fourier Series representation is given by:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

where c_n are the Fourier coefficients, given by:

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\frac{2\pi nt}{T}}\, dt$$

Properties and Significance

1. Spectral Analysis

- Fourier Series decomposes a signal into its constituent frequencies, revealing the signal's frequency components and their respective magnitudes and phases. This is crucial for analyzing signals in frequency domain, which often provides insights not readily apparent in time domain.

2. Orthogonality

- The sinusoidal basis functions in the Fourier Series are orthogonal over one period T. This orthogonality property simplifies the computation of Fourier coefficients and allows for efficient signal analysis and synthesis.

3. Frequency Domain Representation

- By analyzing the Fourier coefficients, we obtain the signal's frequency domain representation, showing the amplitudes and phases of its individual frequency components. This is essential for understanding how a system affects different frequencies within a signal.

4. Signal Reconstruction

- Fourier Series enables the reconstruction of a signal from its frequency components. By summing up sinusoidal functions weighted by their respective Fourier coefficients, we can reconstruct an approximation of the original signal.

5. Signal Analysis

- Fourier Series is used to analyze periodic signals in both time and frequency domains, revealing their spectral characteristics and aiding in filtering, modulation, and demodulation processes.

6. System Analysis

- It helps in understanding how systems affect signals' frequency content, aiding in the design and analysis of filters, amplifiers, and other systems that manipulate signals.

7. Communication Systems

- Fourier Series is fundamental in the analysis and design of communication systems, such as modulation schemes, where signals are modulated onto carrier frequencies for transmission and demodulated at the receiver end.

8. Control Systems

- Fourier Series aids in analyzing system responses to periodic inputs, facilitating stability analysis and controller design in control systems.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Chapter 7: Fourier Transform

1. Forward Fourier Transform

$$F(k) = F_x[f(x)](k) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ikx}dx$$

2. Inverse Fourier Transform

$$f(x) = F_k^{-1}[F(k)](x) = \int_{-\infty}^{\infty} F(k)e^{2\pi ikx}dk$$

3. Fourier Transform in Two Dimensions

$$F(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(k_x,k_y)e^{-2\pi i(k_x x+k_y y)}dk_x dk_y$$

$$f(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(x,y)e^{2\pi i(k_x x+k_y y)}dxdy$$

4. Fourier Sine Transform

$$F_x^{(s)}[f(x)](k) = I[F_x[f(x)](k)]$$
$$F_x^{(s)}[f(x)](k) = \int_{-\infty}^{\infty} sin(2\pi kx)f(x)dx$$

5. Fourier Cosine Transform

$$F_x^{(c)}[f(x)](k) = R[F_x[f(x)](k)]$$
$$F_x^{(c)}[f(x)](k) = \int_{-\infty}^{\infty} cos(2\pi kx)f(x)dx$$

6. Continuous-Time Fourier Transform (CTFT):

For a continuous-time signal $x(t)$, the continuous-time Fourier Transform (CTFT) $X(f)$ is defined as:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$$

where f is the frequency variable and j represents the imaginary unit.

7. Discrete-Time Fourier Transform (DTFT):

For a discrete-time signal $x[n]$, the discrete-time Fourier Transform (DTFT) $X(e^{j\omega})$ is defined as:

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

where ω is the digital frequency variable.

Chapter 8: Z Transform

Definition

- The Z-transform is a mathematical transformation used in the analysis and representation of discrete-time signals and systems. It plays a crucial role in digital signal processing and control theory.
- The Z-transform is a discrete-time analog of the Laplace transform for continuous-time signals. It transforms a discrete-time signal sequence x[n] into a complex function of a complex variable z.

Mathematical Representation

For a discrete-time signal $x[n]$, the Z-transform $X(z)$ is defined as:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

where z is a complex variable.

Properties and Significance

1. Frequency Domain Representation

- The Z-transform provides a frequency domain representation of discrete-time signals and systems, revealing their frequency characteristics and enabling analysis in terms of complex frequencies.

2. Transfer Function

- The Z-transform is used to represent the transfer function of discrete-time systems. It facilitates the analysis of system behavior, stability, and response to input signals.

3. Pole-Zero Analysis

- Similar to the Laplace transform, the Z-transform allows for pole-zero analysis, where poles and zeros in the Z-plane provide insights into system dynamics and frequency response.

4. Inverse Z-transform

- The inverse Z-transform allows for the reconstruction of a discrete-time signal from its Z-transform representation. It enables the conversion of signals from the Z-domain back to the time domain.

5. Digital Signal Processing (DSP)

- The Z-transform is extensively used in DSP for system analysis, filter design, spectral analysis, and digital filter implementation.

6. Control Systems

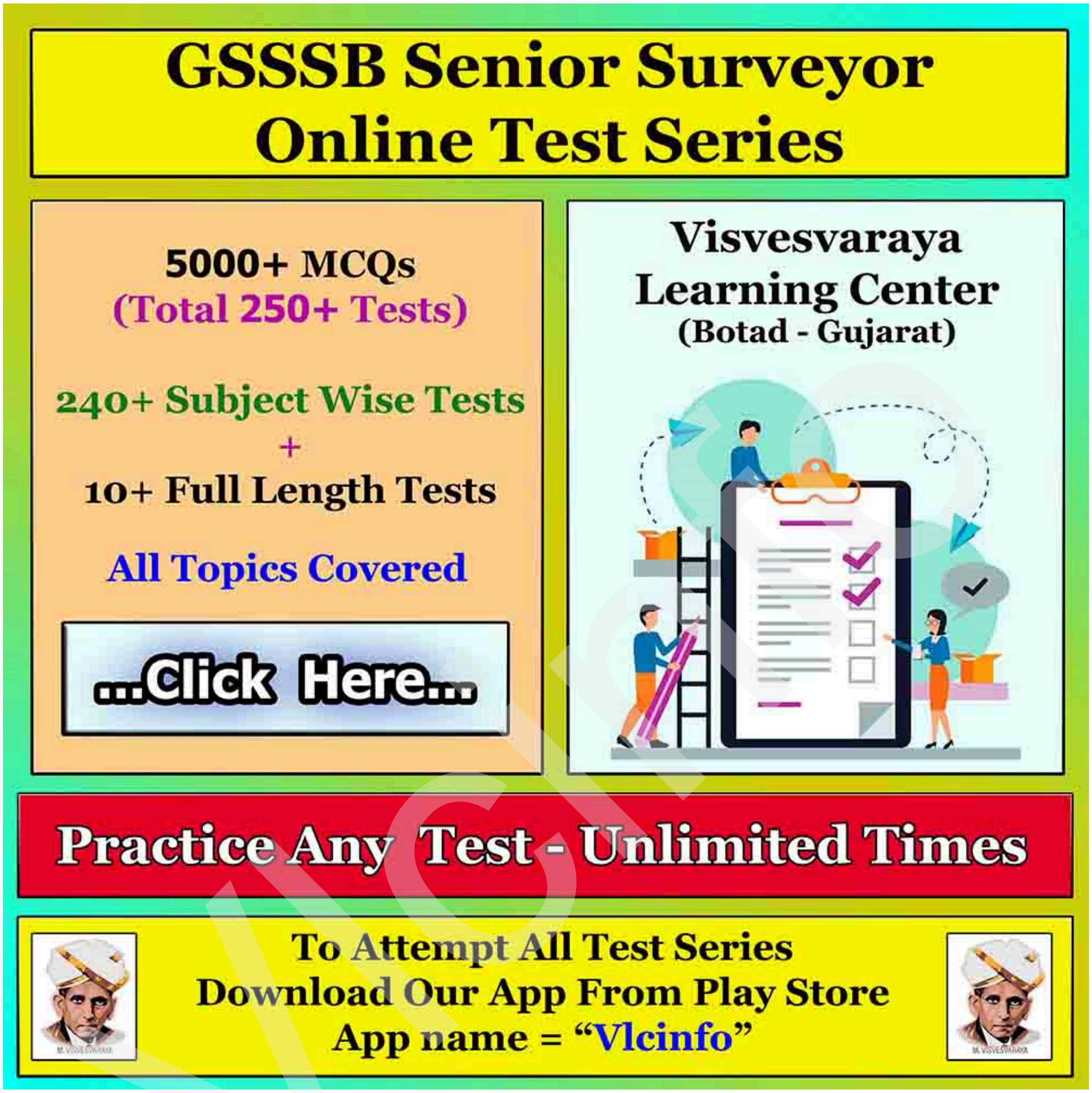
- In control theory, the Z-transform is employed for the analysis and design of discrete-time control systems, including stability analysis, controller design, and system simulation.

7. Communication Systems

- The Z-transform plays a role in the analysis and design of digital communication systems, particularly in signal modulation, demodulation, and equalization.

8. Image Processing

- In image processing, the Z-transform is utilized for image enhancement, filtering, compression, and feature extraction in digital images.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